Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Mathematics

A Further Pure Maths 2 (9FM0/3A)

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Summer 2019

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed

through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

A Further Pure Maths Option Paper 1 (9FM0/3A) 1906

Mark Scheme – Final

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>1</td><td>Step length = 0.4</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>

	y_0	y_1	y_2	y_3	y_4
x	0.4	0.8	1.2	1.6	2
y	$e^{0.16}$	$e^{0.64}$	$e^{1.44}$	$e^{2.56}$	e^{4}
	1.173…	1.896…	4.220…	12.935…	54.598…

</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>$y_0+4y_1+2y_2+4y_3+y_4=123.54....$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>$\int_{0.4}^{2} e^{x^2}\,dx \approx \frac{0.4}{3}\times\{1.173...+54.598...+4(1.896...+12.935...)+2(4.220...)\}$
$\approx \frac{0.4}{3}\times"123.54..."$</td><td>dM1</td><td>1.1b</td></tr>
<tr><td></td><td>$=16.5$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td colspan="4" align="right">(5 marks)</td></tr>
<tr><td colspan="4" align="center">Notes</td></tr>
</table>

B1: Correct step length of 0.4 which may be implied e.g. by their 0.4, 0.8, etc.

M1: Attempts to find y values for their x values – may be in terms of e or numerical values. Must see an attempt to find at least 3 values.

M1: Correct structure for y values of Simpson's rule (ends + 2evens + 4odds) (must have an odd number of ordinates). Must be y values **not** x values.

dM1: $\frac{"0.4"}{3}\times\text{their}\,123.54...$ or for $\frac{h}{3}\times\text{their}\,123.54...$ leading to a value and where h has clearly been defined earlier.

Dependent on both previous method marks

A1: Awrt 16.5

Note that a minimum we would expect to see for full marks is:

$h = 0.4$

	y_0	y_1	y_2	y_3	y_4
x	0.4	0.8	1.2	1.6	2
y	$e^{0.16}$	$e^{0.64}$	$e^{1.44}$	$e^{2.56}$	e^{4}
	1.173…	1.896…	4.220…	12.935…	54.598…

$$A \approx \frac{h}{3}[y_0+4y_1+2y_2+4y_3+y_4]=16.5$$

(Note that a calculator gives 16.030…for the area)

Question	Scheme	Marks	AOs
2	$u = x^3 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 3x^2, \frac{\mathrm{d}^2u}{\mathrm{d}x^2} = 6x, \frac{\mathrm{d}^3u}{\mathrm{d}x^3} = 6$	M1	1.1b
	$v = \sin kx \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = k\cos kx, \frac{\mathrm{d}^2v}{\mathrm{d}x^2} = -k^2\sin kx, \frac{\mathrm{d}^3v}{\mathrm{d}x^3} = -k^3\cos kx,$ $\frac{\mathrm{d}^4v}{\mathrm{d}x^4} = k^4\sin kx, \frac{\mathrm{d}^5v}{\mathrm{d}x^5} = k^5\cos kx$	M1	2.1
	$\frac{\mathrm{d}^5y}{\mathrm{d}x^5} = x^3k^5\cos kx + 5\times 3x^2\times k^4\sin kx + \frac{5\times 4}{2}\times 6x\times\left(-k^3\cos kx\right) +$ $\frac{5\times 4\times 3}{3!}\times 6\times\left(-k^2\sin kx\right)$	M1	2.1
	$= \left(k^2x^2 - 60\right)k^3x\cos kx + 15\left(k^2x^2 - 4\right)k^2\sin kx$	A1	1.1b
		(4)	
		(4 marks)	

Notes

M1: Differentiates $u = x^3$ three times. Need to see $x^3 \to ...x^2 \to ...x \to k$

M1: Uses $v = \sin kx$ to establish the form of the derivatives. Need to see at least alternating $k^{\cdots}\sin kx$ and $k^{\cdots}\cos kx$ with increasing powers of k for at least 3 derivatives.

M1: Uses a correct formula with 2 and 3! (or 6) with terms shown to disappear after the fourth term. This needs to be a correct application of the theorem so that the correct binomial coefficients need to go with the correct pairings of their derivatives. If there is any doubt, at least 3 terms should have the correct structure. Allow equivalent notation for the binomial coefficients e.g. $\binom{5}{0}, \binom{5}{1}$ etc. or ${}^5\mathrm{C}_0, {}^5\mathrm{C}_1$ etc.

A1: Correct expression in the required form with correct values of A, B and C. Apply isw if necessary e.g. if a correct expression is followed by $A = 60$, $B = 15$, $C = -4$
(NB $A = -60$, $B = 15$, $C = -4$)

If there is no use Lebnitz's theorem e.g. repeated differentiation of products, this scores no marks.

Question	Scheme	Marks	AOs
3(a)	$\frac{d^2y}{dx^2}=1-2y\frac{dy}{dx}\Rightarrow\frac{d^3y}{dx^3}=-2y\frac{d^2y}{dx^2}-2\left(\frac{dy}{dx}\right)^2$	M1 A1	1.1b 1.1b
	$\frac{d^4y}{dx^4}=-2\frac{dy}{dx}\frac{d^2y}{dx^2}-2y\frac{d^3y}{dx^3}-4\frac{dy}{dx}\frac{d^2y}{dx^2}=-6\frac{dy}{dx}\frac{d^2y}{dx^2}-2y\frac{d^3y}{dx^3}$	**d**M1	2.1
	$\frac{d^5y}{dx^5}=-6\frac{dy}{dx}\frac{d^3y}{dx^3}-6\left(\frac{d^2y}{dx^2}\right)^2-2y\frac{d^4y}{dx^4}-2\frac{dy}{dx}\frac{d^3y}{dx^3}$ $=-2y\frac{d^4y}{dx^4}-8\frac{dy}{dx}\frac{d^3y}{dx^3}-6\left(\frac{d^2y}{dx^2}\right)^2$	A1	2.1
		(4)	
(b)	$x=0, y=1\Rightarrow\left(\frac{dy}{dx}\right)_0=0-1^2=-1$	B1	2.2a
	$\left(\frac{d^2y}{dx^2}\right)_0=1-2(1)(-1)=3,\left(\frac{d^3y}{dx^3}\right)_0=-2(1)(3)-2(-1)^2=-8$ $\left(\frac{d^4y}{dx^4}\right)_0=-6(-1)(3)-2(1)(-8)=34,$ $\left(\frac{d^5y}{dx^5}\right)_0=-2(1)(34)-8(-1)(-8)-6(3)^2=-186$	M1 A1	1.1b 1.1b
	$y=y(0)+x\left(\frac{dy}{dx}\right)_0x+\frac{x^2}{2!}\left(\frac{d^2y}{dx^2}\right)_0+\frac{x^3}{3!}\left(\frac{d^3y}{dx^3}\right)_0+\frac{x^4}{4!}\left(\frac{d^4y}{dx^4}\right)_0+\frac{x^5}{5!}\left(\frac{d^5y}{dx^5}\right)_0+...$ With their values	M1	2.5
	$(y=)1-x+\frac{3}{2}x^2-\frac{8}{6}x^3+\frac{34}{24}x^4-\frac{186}{120}x^5+...$ $(y=)1-x+\frac{3}{2}x^2-\frac{4}{3}x^3+\frac{17}{12}x^4-\frac{31}{20}x^5+...$	A1ft	1.1b
		(5)	
		(9 marks)	

Notes

(a)

M1: Attempts to find the second and third derivatives:

This requires $\frac{d^2y}{dx^2}=1\pm2y\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}=\pm2y\frac{dy}{dx}$ followed by $\frac{d^3y}{dx^3}=\pm2y\frac{d^2y}{dx^2}\pm...$ or $\frac{d^3y}{dx^3}=\pm...\pm2\left(\frac{dy}{dx}\right)^2$

A1: Correct second **and** third derivatives.

dM1: Continues to differentiate to reach the 5th derivative. **This is dependent on the first method mark but there is no need to check the detail and the mark can be awarded as long as the 5th derivative is reached.**

A1: Completes the process, collecting terms if necessary, to obtain the correct expression (NB $a=-2$, $b=-8$, $c=-6$)

Allow dash/dot notation for the derivatives but the final answer must be in the correct form.

Note that if $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \pm 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ is obtained initially, allow a full recovery in (a).

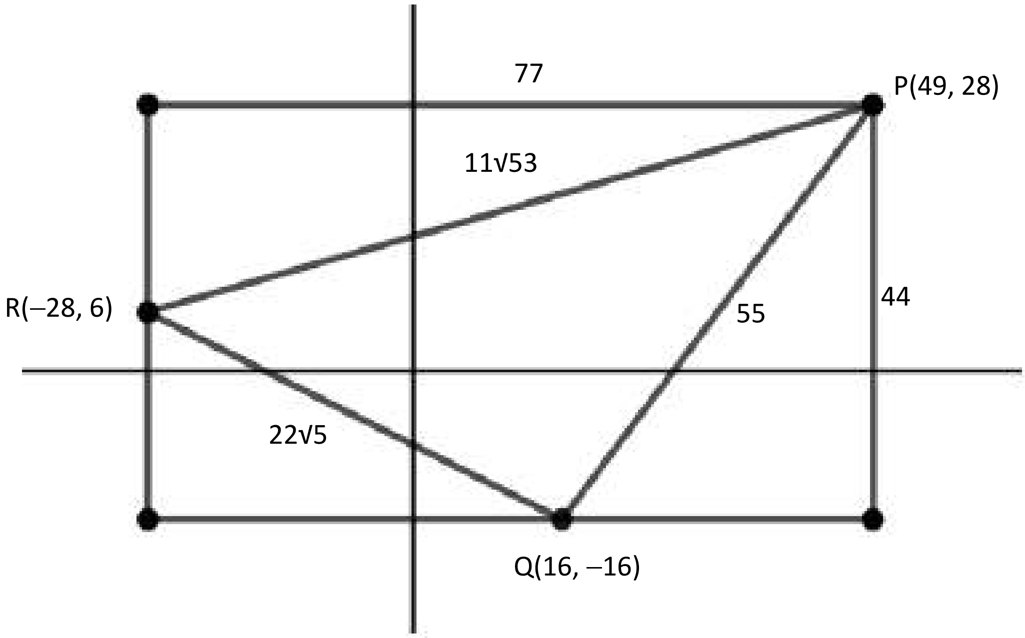
Note that (a) can be found using Leibnitz's theorem and the following scheme should be applied:
M1: $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 1 \pm 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \pm 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ followed by an attempt to differentiate y 3 times and $\frac{\mathrm{d}y}{\mathrm{d}x}$ 3 times.
A1: All correct
dM1: $\frac{\mathrm{d}^5y}{\mathrm{d}x^5} = -2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^4y}{\mathrm{d}x^4} - 3\times 2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^3y}{\mathrm{d}x^3} - 3\times 2\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^2 - 2y\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\frac{\mathrm{d}y}{\mathrm{d}x}$ (correct application of Leibnitz)
A1: $= -2y\frac{\mathrm{d}^4y}{\mathrm{d}x^4} - 8\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^3y}{\mathrm{d}x^3} - 6\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^2$
As in the main scheme, if $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \pm 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ is obtained initially, allow a full recovery in (a).
Alternative for (a):
M1: $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 1 - 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - 2y\left(x - y^2\right) = 1 - 2xy + 2y^3 \Rightarrow \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = -2y - 2x\frac{\mathrm{d}y}{\mathrm{d}x} + 6y^2\frac{\mathrm{d}y}{\mathrm{d}x}$
Score for the second derivative form as in the main scheme and then an attempt at the third derivative with at least 2 terms correct.
A1: Fully correct
Then as main scheme.
(b)
B1: Deduces the correct value for $y'(0)$
M1: Finds the values of all the other derivatives at $x = 0$ up to 5th. There is no need to check their values as long as there is no obvious incorrect work, but values for all the derivatives up to the 5th must be found
A1: All values correct (as single values – e.g. do not allow unsimplified)
M1: Applies the correct Maclaurin series for their values including the factorials up to the term in x^5
A1ft: Correct expansion, follow through their values for the derivatives. This does not have to be simplified but the factorials need to be evaluated. **Once a correct, or correct follow through expression is seen apply isw.**

uestion	Scheme	Marks	AOs
4	$y^2=16x \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}=16 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8}{y}=\frac{8}{4p}$ Requires $\alpha y\frac{\mathrm{d}y}{\mathrm{d}x}=\beta \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{f}\left(p\,or\,q\right)$ $y^2=16x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2x^{-\frac{1}{2}}=2\left(p^2\right)^{-\frac{1}{2}}$ Requires $\frac{\mathrm{d}y}{\mathrm{d}x}=\alpha x^{-\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{f}\left(p\,or\,q\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}p}\frac{\mathrm{d}p}{\mathrm{d}x}=\frac{4}{2p}$ Requires $\frac{\mathrm{d}y}{\mathrm{d}x}=their\frac{\mathrm{d}y}{\mathrm{d}p}\div their\frac{\mathrm{d}x}{\mathrm{d}p} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{f}\left(p\,or\,q\right)$	M1	3.1a
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8}{4p} \Rightarrow y-4p=\frac{2}{p}\left(x-p^2\right)$ or $y-4q=\frac{2}{q}\left(x-q^2\right)$	M1 A1	3.1a 1.1b
	Using $x=-28$ and $y=6$, $6p=-56+2p^2 \Rightarrow p=\ldots$	M1	3.1a
	Alternative for 3rd Method mark: $py=2x+2p^2,\ qy=2x+2q^2 \Rightarrow x=pq, y=2\left(p+q\right)$ Using $x=-28$ and $y=6 \Rightarrow p\left(\text{or}\,q\right)=\ldots$		
	p (or q) $=-4, 7$	A1	1.1b
	$\left(16,-16\right),\ \left(49,28\right)$	A1	2.2a
	Way 1 $\frac{1}{2}\begin{vmatrix}-28 & 16 & 49 & -28\\ 6 & -16 & 28 & 6\end{vmatrix}=\frac{1}{2}\left\|448+448+294-96+784+784\right\|$	M1	3.1a
	Way 2 $77\times44-\frac{1}{2}\times44\times22-\frac{1}{2}\times77\times22-\frac{1}{2}\times44\times33$		
	Way 3 $\frac{1}{2}22\sqrt{5}\times11\sqrt{53}\sin\left(\cos^{-1}\left(\frac{\left(11\sqrt{53}\right)^2+\left(22\sqrt{5}\right)^2-55^2}{2\times11\sqrt{53}\times22\sqrt{5}}\right)\right)$ NB angle at R is 42.5 (1dp)		
	Way 4 $\frac{1}{2}55\times11\sqrt{53}\sin\left(\cos^{-1}\left(\frac{\left(11\sqrt{53}\right)^2+55^2-\left(22\sqrt{5}\right)^2}{2\times11\sqrt{53}\times55}\right)\right)$ NB angle at P is 37.2 (1dp)		
	Way 5 $\frac{1}{2}55\times22\sqrt{5}\sin\left(\cos^{-1}\left(\frac{\left(22\sqrt{5}\right)^2+55^2-\left(11\sqrt{53}\right)^2}{2\times22\sqrt{5}\times55}\right)\right)$ NB angle at Q is 100.3 (1dp)		

	Way 6 $S=\dfrac{55+22\sqrt{5}+11\sqrt{53}}{2}\Rightarrow A=\sqrt{S\left(S-55\right)\left(S-22\sqrt{5}\right)\left(S-11\sqrt{53}\right)}$		
	Way 7 Line PR $y-28=\dfrac{28-6}{49+28}\left(x-49\right), x=16\Rightarrow y=\dfrac{130}{7}$ $A=\dfrac{1}{2}\times\dfrac{242}{7}\left(28+16\right)+\dfrac{1}{2}\times\dfrac{242}{7}\left(49-16\right)$		
	Way 8 $\dfrac{1}{2}\left\|RP\times QP\right\|=\dfrac{1}{2}\left\|\begin{pmatrix}77\\22\end{pmatrix}\times\begin{pmatrix}33\\44\end{pmatrix}\right\|=\dfrac{1}{2}\left(2662\right)$ For such methods, a minimum of e.g. $\dfrac{1}{2}\left(2662\right)$ must be seen		
	= 1331 (units^2)*	A1*	1.1b
		(8)	
			(8 marks)

Notes

M1: Attempts to solve the problem by using differentiation to obtain an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of p or q.
See scheme for requirements for this mark depending on the method chosen.
(Can be implied by a correct expression)
M1: Correct straight line method to find the equation of the tangent using P or Q.
If using $y = mx + c$, must reach as far as $c = \ldots$
A1: Obtains a correct general tangent at P or Q or both
Note that if a correct tangent equation is quoted, the first 3 marks are available
M1: Uses $x = -28$ and $y = 6$ **with the values correctly placed** in one of their tangent equations **and** attempts to solve the resulting 3TQ to obtain 2 values for p (or q).
An **alternative approach** for this mark is to obtain equations for both tangents and solve simultaneously to obtain the coordinates for the intersection and then to use $x = -28$ and $y = 6$ to find values for p and q. Note that a calculator may be used for the simultaneous equations but answers must be correct for their equations if no working is shown.
A1: Correct values
A1: Deduces the correct coordinates of P and Q
M1: Completes the problem by using a suitable complete correct method for finding the area of PQR – See examples – there will be others – in general, score M1 for a correct triangle area method for their values
A1*: Correct area. Allow this mark even if the candidate reverts to decimals within their solution, providing all the working is correct.



Generally, using midpoints of sides is unlikely to be successful, however, the line from R to the midpoint of PQ is horizontal so this is a correct approach:

$$\text{Midpoint: } \left(\frac{49+16}{2}, \frac{28-16}{2}\right) = \left(\frac{65}{2}, 6\right) \Rightarrow \text{Area} = \frac{1}{2}\left(\frac{65}{2}+28\right)\times 44 = 1331$$

Question	Scheme	Marks	AOs
5	$4\cos x - 3\sin x = 4\left(\frac{1-t^2}{1+t^2}\right) - 3\left(\frac{2t}{1+t^2}\right)$	B1	1.1a
	$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1+t^2}{2}$ or $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{2}{1+t^2}$ or $\mathrm{d}x = \frac{2\mathrm{d}t}{1+t^2}$ or $\mathrm{d}t = \frac{1+t^2}{2}\mathrm{d}x$ oe	B1 **M1 on ePEN**	2.1
	$\int \frac{1}{4\cos x - 3\sin x}\mathrm{d}x = \int \frac{1}{4\left(\frac{1-t^2}{1+t^2}\right) - 3\left(\frac{2t}{1+t^2}\right)} \times \frac{2\mathrm{d}t}{1+t^2}$	M1	2.1
	$= \int \frac{2}{4-4t^2-6t}(\mathrm{d}t)$ or $\int \frac{1}{2-2t^2-3t}(\mathrm{d}t)$ or $\int \frac{-1}{2t^2+3t-2}(\mathrm{d}t)$ etc.	A1	1.1b
	$\frac{-2}{4t^2+6t-4} = \frac{-1}{(t+2)(2t-1)} = \frac{A}{(t+2)} + \frac{B}{(2t-1)}$ $\frac{-1}{(t+2)(2t-1)} = \frac{1}{5(t+2)} + \frac{2}{5(1-2t)}$	M1	3.1a
	$\Rightarrow I = \frac{1}{5}\int \frac{1}{(t+2)} - \frac{2}{(2t-1)}(\mathrm{d}t)$ or equivalent	A1	1.1b
	$= \frac{1}{5}\int \frac{1}{(t+2)} - \frac{2}{(2t-1)}\mathrm{d}t = \frac{1}{5}\ln(t+2) - \frac{1}{5}\ln(1-2t)(+k)$	A1	1.1b
	$= \frac{1}{5}\ln\left(\frac{2+t}{1-2t}\right)(+k) = \frac{1}{5}\ln\left(\frac{2+\tan\left(\frac{x}{2}\right)}{1-2\tan\left(\frac{x}{2}\right)}\right) + k$ *	A1*	2.1
		(8)	
	Alternative for final 4 marks:		
	$= \int \frac{2}{4-4t^2-6t}(\mathrm{d}t) = -\frac{1}{2}\int \frac{1}{t^2+\frac{3}{2}t-1}(\mathrm{d}t) = -\frac{1}{2}\int \frac{1}{\left(t+\frac{3}{4}\right)^2 - \frac{25}{16}}(\mathrm{d}t)$ or e.g. $\int \frac{1}{\frac{25}{8} - 2\left(t+\frac{3}{4}\right)^2}(\mathrm{d}t)$	M1 A1	3.1a 1.1b
	$-\frac{1}{2} \times \frac{1}{2} \times \frac{4}{5}\ln\left(\frac{t+\frac{3}{4}-\frac{5}{4}}{t+\frac{3}{4}+\frac{5}{4}}\right)(+c)$	A1	1.1b
	$-\frac{1}{5}\ln\left\lvert\frac{\tan\left(\frac{x}{2}\right)-\frac{1}{2}}{\tan\left(\frac{x}{2}\right)+2}\right\rvert + c = \frac{1}{5}\ln\left\lvert\frac{\tan\left(\frac{x}{2}\right)+2}{\tan\left(\frac{x}{2}\right)-\frac{1}{2}}\right\rvert + c = \frac{1}{5}\ln\left(\frac{\tan\left(\frac{x}{2}\right)+2}{\frac{1}{2}-\tan\left(\frac{x}{2}\right)}\right) + c$ $= \frac{1}{5}\ln\left(\frac{2\left(\tan\left(\frac{x}{2}\right)+2\right)}{1-2\tan\left(\frac{x}{2}\right)}\right) + c = \frac{1}{5}\ln\left(\frac{\left(\tan\left(\frac{x}{2}\right)+2\right)}{1-2\tan\left(\frac{x}{2}\right)}\right) + \frac{1}{5}\ln 2 + c$ $= \frac{1}{5}\ln\left(\frac{\left(\tan\left(\frac{x}{2}\right)+2\right)}{1-2\tan\left(\frac{x}{2}\right)}\right) + k$	A1*	2.1
		(8 marks)	

Notes
B1: Uses the **correct** formulae to express $4\cos x - 3\sin x$ in terms of t B1(M1 on ePEN): Correct equation in terms of dx, dt and t – can be implied if seen as part of their substitution. M1: Makes a **complete** substitution to obtain an integral in terms of t only. Allow slips with the substitution of "dx" but must be dx = f(t)dt where f(t) ≠ 1. This mark is also available if the candidate makes errors when attempting to simplify $4\left(\frac{1-t^2}{1+t^2}\right)-3\left(\frac{2t}{1+t^2}\right)$ before attempting the substitution. A1: For obtaining a fully correct simplified integral with a constant in the numerator and a 3 term quadratic expression in the denominator. ("dt" not required) M1: Realises the need to express the integrand in terms of partial fractions in order to attempt the integration. **Must have a 3 term quadratic expression in the denominator and a constant in the numerator.** A1: Correct integral in terms of partial fractions – allow any equivalent **correct** integral. ("dt" not required) A1: Fully correct integration in terms of t A1*: Correct solution with no errors including "+ k" (allow "+ c") and with the constant dealt with correctly if necessary. The denominator must also be dealt with correctly. E.g. if it appears as $2t - 1$ initially and becomes $1 - 2t$ without justification, this final mark should be withheld. **<u>Alternative for final 4 marks:</u>** M1: Realises the need to express the integrand in completed square form in order to attempt the integration. **Must have a 3 term quadratic expression in the denominator and a constant in the numerator.** A1: Correct integral with the square completed – allow any equivalent **correct** integral ("dt" not required) A1: Fully correct integration in terms of t A1*: Correct solution with no errors including "+ k" (allow "+ c") and with the constant dealt with correctly if necessary as shown in the scheme and with the denominator dealt with correctly if necessary. Note that it is acceptable for the "dt" to appear and disappear throughout the proof as long as the intention is clear.

Question	Scheme	Marks	AOs
6(a)	**Examples:** $t=\mathrm{e}^x \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}C}=\mathrm{e}^x\frac{\mathrm{d}x}{\mathrm{d}C}$ or $\frac{\mathrm{d}C}{\mathrm{d}x}=t\frac{\mathrm{d}C}{\mathrm{d}t}$ or $\frac{\mathrm{d}C}{\mathrm{d}t}=\mathrm{e}^{-x}\frac{\mathrm{d}C}{\mathrm{d}x}$ or $\frac{\mathrm{d}C}{\mathrm{d}t}=\frac{1}{t}\frac{\mathrm{d}C}{\mathrm{d}x}$	M1	1.1b
	E.g. $\frac{\mathrm{d}C}{\mathrm{d}x}=t\frac{\mathrm{d}C}{\mathrm{d}t}\Rightarrow\frac{\mathrm{d}^2C}{\mathrm{d}x^2}\times\frac{\mathrm{d}x}{\mathrm{d}t}=t\frac{\mathrm{d}^2C}{\mathrm{d}t^2}+\frac{\mathrm{d}C}{\mathrm{d}t}$	**d**M1 A1	2.1 1.1b
	$\frac{\mathrm{d}^2C}{\mathrm{d}x^2}\times\frac{1}{t}=t\frac{\mathrm{d}^2C}{\mathrm{d}t^2}+\frac{1}{t}\frac{\mathrm{d}C}{\mathrm{d}x}\Rightarrow t^2\frac{\mathrm{d}^2C}{\mathrm{d}t^2}=\frac{\mathrm{d}^2C}{\mathrm{d}x^2}-\frac{\mathrm{d}C}{\mathrm{d}x}$ $t^2\frac{\mathrm{d}^2C}{\mathrm{d}t^2}-5t\frac{\mathrm{d}C}{\mathrm{d}t}+8C=\frac{\mathrm{d}^2C}{\mathrm{d}x^2}-\frac{\mathrm{d}C}{\mathrm{d}x}-5\frac{\mathrm{d}C}{\mathrm{d}x}+8C$	**d**M1	2.1
	$\frac{\mathrm{d}^2C}{\mathrm{d}x^2}-6\frac{\mathrm{d}C}{\mathrm{d}x}+8C=\mathrm{e}^{3x}$ *	A1*	1.1b
		(5)	
	<u>Mark (b) and (c) together and ignore labelling</u>		
(b)	$m^2-6m+8=0\Rightarrow m=2,4$	M1	1.1b
	$(C=)\,A\mathrm{e}^{4x}+B\mathrm{e}^{2x}$	A1ft	1.1b
	PI is $C=k\mathrm{e}^{3x}$	B1	2.2a
	$\frac{\mathrm{d}C}{\mathrm{d}x}=3k\mathrm{e}^{3x},\frac{\mathrm{d}^2C}{\mathrm{d}x^2}=9k\mathrm{e}^{3x}\Rightarrow 9k-18k+8k=1\Rightarrow k=-1$	M1	1.1b
	$C=A\mathrm{e}^{4x}+B\mathrm{e}^{2x}-\mathrm{e}^{3x}$	A1	1.1b
	$t=\mathrm{e}^x\Rightarrow C=...$	M1	3.4
	$C=At^4+Bt^2-t^3$	A1	2.2a
		(7)	
(c)	$t=6,C=0\Rightarrow 1296A+36B-216=0$	M1	3.4
	$\frac{\mathrm{d}C}{\mathrm{d}t}=4At^3+2Bt-3t^2\Rightarrow -36=864A+12B-108$	M1	3.4
	$A=0,B=6\Rightarrow C=6t^2-t^3$	A1	1.1b
	$\frac{\mathrm{d}C}{\mathrm{d}t}=12t-3t^2=0\Rightarrow t=4\Rightarrow C=...$	**dd**M1	1.1b
	$C=6(4)^2-(4)^3=32\,\mu\mathrm{gL}^{-1}$	A1	3.2a
		(5)	
			(17 marks)

Notes

(a)

M1: Uses t = e^x to obtain a correct equation in terms of $\frac{\mathrm{d}C}{\mathrm{d}x}$, $\frac{\mathrm{d}C}{\mathrm{d}t}$ and $t\,(\text{or } \mathrm{e}^x)$ or their reciprocals

dM1: Differentiates again **correctly** with the product rule and chain rule in order to obtain an equation involving $\frac{\mathrm{d}^2C}{\mathrm{d}t^2}$ and $\frac{\mathrm{d}^2C}{\mathrm{d}x^2}$. **This needs to be fully correct calculus work allowing sign errors only.**
A1: Correct equation.
dM1: Shows clearly their substitution into the differential equation (or equivalent work) in order to form the new equation. **Dependent on the first method mark and dependent on having obtained two terms for the second derivative.**
Allow substitution for $\frac{\mathrm{d}C}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2C}{\mathrm{d}x^2}$ into equation (II) to achieve equation (I)
A1*: Fully correct proof with no errors
(b)
M1: Forms and solves a quadratic auxiliary equation $m^2 - 6m + 8 = 0$
A1ft: Correct form for the CF for their AE solutions **which must be distinct and real**
B1: Deduces the correct form for the PI ($k\mathrm{e}^{3x}$)
M1: Differentiates their PI, **which is of the correct form**, and substitutes their derivatives into the DE to find "*k*"
A1: Correct GS for *C* in terms of *x* (**this must be seen explicitly unless implied by subsequent work**)
M1: Links the solution to DE (II) to the solution of the model to find the concentration at time *t*
A1: Deduces the correct GS for the concentration
If a correct GS is fortuitously found in (b) (e.g. from an incorrect PI form, allow full recovery in (c).
(c)
M1: Uses the conditions of the model ($t = 6$, $C = 0$) to form an equation in *A* and *B*.
*******Note that **is** acceptable to use their *C* in terms of *x* for this mark as long as they use $x = \ln 6$ when $C = 0$
M1: Uses the conditions of the model $\left(t = 6, \frac{\mathrm{d}C}{\mathrm{d}t} = -36\right)$ to form another equation in *A* and *B*.
*******Note that it is **not** acceptable to use $\frac{\mathrm{d}C}{\mathrm{d}x} = -36$ with $x = \ln 6$, as it is necessary to use $\frac{\mathrm{d}C}{\mathrm{d}t} = \frac{\mathrm{d}C}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}$ e.g. $-36 = \left(4A\mathrm{e}^{4\ln 6} + 2B\mathrm{e}^{2\ln 6} - 3\mathrm{e}^{3\ln 6}\right) \times \mathrm{e}^{-\ln 6}$ or $-216 = 4A\mathrm{e}^{4\ln 6} + 2B\mathrm{e}^{2\ln 6} - 3\mathrm{e}^{3\ln 6}$
A1: Correct equation connecting *C* with *t*
ddM1: Uses a suitable method to find the maximum concentration. E.g. solves $\frac{\mathrm{d}C}{\mathrm{d}t} = 0$ for *t* and substitutes to find *C*. Allow a solution that solves $\frac{\mathrm{d}C}{\mathrm{d}x} = 0$ for *x* and uses this correctly to find *C*.
Dependent on both previous method marks.
A1: Obtains 32 $\mu\mathrm{g L}^{-1}$ using the model. Units are required but allow e.g.

- micrograms per litre
- μg/L
- μg/*l*
- $\mu\mathrm{g}\,l^{-1}$

Question	Scheme	Marks	AOs
7(a)	Examples: Area $APQC$ = Area ABC – Area PBQ Area $APQC$ = Area APC + Area CPQ Area $APQC$ = Area APQ + Area AQC Area $APQC = \frac{1}{2}\lvert\mathbf{AQ}\times\mathbf{PC}\rvert$	M1	3.1a
	Line AB: $r = \begin{pmatrix}3\\4\\5\end{pmatrix} + \lambda\begin{pmatrix}10-3\\-1-4\\5-5\end{pmatrix} = \begin{pmatrix}3\\4\\5\end{pmatrix} + \lambda\begin{pmatrix}7\\-5\\0\end{pmatrix}$ **or** Line BC: $r = \begin{pmatrix}10\\-1\\5\end{pmatrix} + \mu\begin{pmatrix}10-4\\-1-7\\5+9\end{pmatrix} = \begin{pmatrix}10\\-1\\5\end{pmatrix} + \mu\begin{pmatrix}6\\-8\\14\end{pmatrix}$	M1	3.1a
	$4(3+7\lambda)-8(4-5\lambda)+5=2 \Rightarrow \lambda = ... \Rightarrow P$ is … **or** $4(10+6\mu)-8(-1-8\mu)+5+14\mu=2 \Rightarrow \mu = ... \Rightarrow Q$ is … $\left(\text{NB } \lambda = \frac{1}{4},\ \mu = -\frac{1}{2}\right)$	M1	2.1
	$P(4.75,\ 2.75,\ 5)$ **and** $Q(7,\ 3,\ -2)$	A1	1.1b
	Area $ABC = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\7 & -5 & 0\\6 & -8 & 14\end{vmatrix} = \frac{1}{2}\sqrt{70^2+98^2+26^2}$ Area $PBQ = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\5.25 & -3.75 & 0\\3 & -4 & 7\end{vmatrix} = \frac{1}{2}\sqrt{26.25^2+36.75^2+9.75^2}$ Area $APQC = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-4 & 1 & 7\\0.75 & -4.25 & 14\end{vmatrix} = \frac{1}{2}\sqrt{43.75^2+61.25^2+16.25^2}$ NB: Area APQ = 7.7004, Area AQC = 30.8018, Area CPQ = 23.101, Area APC = 15.4008	M1	2.1
	Area ABC – Area PBQ = 38.5*	A1*	1.1b
		(6)	
(b)	$\overrightarrow{AB} = \begin{pmatrix}7\\-5\\0\end{pmatrix},\ \overrightarrow{AC} = \begin{pmatrix}1\\3\\-14\end{pmatrix}, \overrightarrow{AD} = \begin{pmatrix}k-3\\0\\-6\end{pmatrix}$	M1	3.1a

	$\overrightarrow{AB}\times\overrightarrow{AC}\bullet\overrightarrow{AD}=\begin{vmatrix}7 & -5 & 0\\ 1 & 3 & -14\\ k-3 & 0 & -6\end{vmatrix}=...$		
	$\overrightarrow{AB}\times\overrightarrow{AC}\bullet\overrightarrow{AD}=7\times-18+5(-6+14k-42)$	A1	1.1b
	$7\times-18+5(-6+14k-42)=\pm226\Rightarrow k=...$	**d**M1	3.1a
	$k=2$ or $\dfrac{296}{35}$	A1	1.1b
		(4)	
(10 marks)			

Notes

(a)

M1: Identifies a correct strategy to determine the area of the required quadrilateral. The attempt does **<u>not</u> need to be complete** for this mark so one of the statements (or intentions) in the markscheme would be sufficient.

M1: Correct attempt to find the equation of the line *AB* **or** the line *BC*

M1: Uses **at least one** of their lines and the equation of the given plane to determine the value of **at least one** of the parameters and hence the coordinates of *P* **or** *Q*

A1: **Both** coordinates correct – allow as vectors and may be implied if for example the candidate calculates the vectors e.g. AP, AQ, CP, CQ without stating the coordinates explicitly

M1: Uses all the required information to **<u>calculate appropriate areas correctly</u>** leading to the area of the quadrilateral. Needs to be a complete method here.

A1*: Reaches 38.5 with no errors

(b)

M1: Adopts a correct strategy by finding suitable vectors and forming the scalar triple product. This is often done in 2 steps e.g.

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}70\\98\\26\end{pmatrix}\text{ or }\overrightarrow{AB}\times\overrightarrow{AD}=\begin{pmatrix}30\\42\\5k-15\end{pmatrix}\text{ or }\overrightarrow{AC}\times\overrightarrow{AD}=\begin{pmatrix}-18\\48-14k\\-3k+9\end{pmatrix}$$

$$\overrightarrow{AB}\times\overrightarrow{AC}.\overrightarrow{AD}=\begin{pmatrix}70\\98\\26\end{pmatrix}.\begin{pmatrix}k-3\\0\\-6\end{pmatrix}=70k-210-156$$

$$\text{or }\overrightarrow{AB}\times\overrightarrow{AD}.\overrightarrow{AC}=\begin{pmatrix}30\\42\\5k-15\end{pmatrix}.\begin{pmatrix}1\\3\\-14\end{pmatrix}=30+126-70k+210$$

$$\text{or }\overrightarrow{AC}\times\overrightarrow{AD}.\overrightarrow{AB}=\begin{pmatrix}-18\\48-14k\\-3k+9\end{pmatrix}.\begin{pmatrix}7\\-5\\0\end{pmatrix}=-126+70k-240$$

If it is not clear that the vector product is being used, at least 2 of the components should be correct.

A1: Correct expression for the triple product in terms of *k* (should be $\pm(70k-366)$)

Ignore the presence or absence of "1/6" for the first 2 marks
dM1: Realises that ±226 is possible for the value of the triple product and attempts to solve to obtain 2 values for *k*. **Dependent on the previous method mark.**
A1: Correct values (must be exact)

Question	Scheme	Marks	AOs
8(a)	$\frac{x^2}{16}-\frac{y^2}{9}=1\Rightarrow\frac{x}{8}-\frac{2yy'}{9}=0\Rightarrow y'=\frac{9x}{16y}=\frac{36\cosh\theta}{48\sinh\theta}$ or $x=4\cosh\theta, y=3\sinh\theta\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3\cosh\theta}{4\sinh\theta}$	M1	3.1a
	$y-3\sinh\theta=\frac{3\cosh\theta}{4\sinh\theta}(x-4\cosh\theta)$	M1	3.1a
	$y=0\Rightarrow x=\frac{4}{\cosh\theta}$	A1	2.2a
	line l_2 has equation $x = 4$	B1	2.2a
	$x=4\Rightarrow y-3\sinh\theta=\frac{3\cosh\theta}{4\sinh\theta}(4-4\cosh\theta)$	M1	2.1
	$y=\frac{3\cosh\theta-3}{\sinh\theta}$	A1	2.2a
	M is $\left(\frac{1}{2}\left(4+\frac{4}{\cosh\theta}\right),\frac{1}{2}\left(\frac{3\cosh\theta-3}{\sinh\theta}\right)\right)$	M1	1.1b
	$x=2+\frac{2}{\cosh\theta}\Rightarrow\cosh\theta=\frac{2}{x-2}$ $\Rightarrow y^2=\frac{9(\cosh\theta-1)^2}{4\sinh^2\theta}=\frac{9\left(\frac{2}{x-2}-1\right)^2}{4\left(\left(\frac{2}{x-2}\right)^2-1\right)}$	M1	3.1a
	$=\frac{9\left(\frac{2}{x-2}-1\right)^2}{4\left(\frac{2}{x-2}-1\right)\left(\frac{2}{x-2}+1\right)}=\frac{9\left(\frac{2}{x-2}-1\right)}{4\left(\frac{2}{x-2}+1\right)}=\frac{9(4-x)}{4x}$ *	A1*	1.1b
	Alternative for M1A1: $y^2=\frac{9(\cosh\theta-1)^2}{4\sinh^2\theta}=\frac{9(\cosh\theta-1)^2}{4(\cosh\theta-1)(\cosh\theta+1)}=\frac{9(\cosh\theta-1)}{4(\cosh\theta+1)}$ $\frac{9(4-x)}{4x}=\frac{9\left(4-2-\frac{2}{\cosh\theta}\right)}{8+\frac{8}{\cosh\theta}}=\frac{9(\cosh\theta-1)}{4(\cosh\theta+1)}\Rightarrow y^2=\frac{9(4-x)}{4x}$		
	$p = 2$ **or** $q = 4$	M1	3.1a
	$p = 2$ **and** $q = 4$	A1	1.1b
		(11)	

(b)	$b^2 = a^2\left(e^2-1\right) \Rightarrow 9 = 16\left(e^2-1\right) \Rightarrow e = \frac{5}{4}$ Focus is at $x = ae = 4\times\frac{5}{4} = 5$	M1	1.1b
	$d >$ "5" $- 4 = \ldots$	M1	3.1a
	$d > 1$*	A1*	1.1b
		(3)	
			(14 marks)

Notes

(a)

M1: Attempts to solve the problem by using differentiation to obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. Allow this mark for $\frac{x^2}{16}-\frac{y^2}{9}=1 \Rightarrow \alpha x - \beta y y' = 0 \Rightarrow y' = \ldots$ or an attempt to differentiate x and y wrt θ and then $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}\theta} \div \frac{\mathrm{d}x}{\mathrm{d}\theta} = \ldots$

M1: Correct straight line method using the coordinates of P and their gradient in terms of θ

Allow the results for the first 2 M marks to be "quoted", but any statements must be correct to score the marks.

A1: Uses $y = 0$ to deduce the correct coordinates (or value of x) for the point A. Allow in any form, simplified or unsimplified (e.g. unsimplified: $x = 4\cosh\theta - 4\tanh\theta\sinh\theta$)

B1: Deduces that the equation of l_2 is $x = 4$ (may be implied by $x = 4$ used to find y coordinate of B)

M1: Realises that $x = 4$ is all that is needed for the second line and substitutes this into the first line in order to find the point B

A1: Deduces the correct coordinates or y value for B

(e.g. unsimplified $y = \frac{3}{\tanh\theta} - \frac{3\cosh\theta}{\tanh\theta} + 3\sinh\theta$)

M1: Uses a correct method for the midpoint of AB (coordinates must be the right way round). This may be seen as the coordinates written separately e.g. $x = \ldots, y = \ldots$

M1: Having found the midpoint, identifies a correct strategy that will enable a Cartesian equation to be found. E.g. find $\cosh\theta$ in terms of x and substitutes into y or y^2 to obtain an equation in terms of y and x only. Mark positively here, so allow the mark if the candidate makes progress in eliminating θ even if there are slips in the working.

A1*: Obtains the printed answer with no errors

Alternative for the previous 2 marks: Substitutes the coordinates of their midpoint into both sides of the given equation in an attempt to show they are equal. Again mark positively but having made the substitution, some progress needs to be made in showing that both sides are equal. **For this method there must be a minimal conclusion for the A1 e.g. tick, hence true etc.**

Note that these 2 marks can also be attempted by expressing the midpoint in terms of exponentials – if you are in doubt whether to award marks seek advice from your Team Leader.

M1: For $p = 2$ **or** $q = 4$

A1: For $p = 2$ **and** $q = 4$

(b)
M1: A complete method for finding the *x* coordinate of the focus using a correct eccentricity formula to find a value for e and then calculating 4e
M1: Completes the problem by subtracting 4 from the *x* coordinate of the focus
A1*: Correct answer
If you come across correct attempts using Pythagoras to prove the result send to review.

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